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ABSTRACT

The Welch-James (WJ) and Improved General Approximation (IGA) tests for the within-subjects main and interaction effects in a split-plot repeated measurement design were investigated when least squares estimates and robust estimates based on trimmed means were used. Variables manipulated in the Monte Carlo study included the degree of multivariate normality, degree of departure from the assumption of multisample sphericity, total sample size, degree of sample size imbalance, and number of levels of within-subjects factor. Consistent with previous research, the WJ and IGA procedures based on least squares estimates were not always robust to violations of the multisample sphericity assumption when the data were obtained from multivariate nonnormal distributions. Adoption of trimmed mean estimators resulted in dramatic improvements in the Type I error performance of the WJ procedure. These results suggest that it is possible to obtain tests of within-subjects effects in split-plot designs that are robust to the combined effects of assumption violations. (Contains 6 tables and 21 references.) (Author/SLD)

Trimmed Means in Split-Plot Repeated Measurement Designs

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Abstract

The Welch-James (WJ) and Improved General Approximation (IGA) tests for the within-subjects main and interaction effects in a split-plot repeated measurement design were investigated when least squares estimates and robust estimates based on trimmed means were used. Variables manipulated in the study included the degree of multivariate nonnormality, degree of departure from the assumption of multisample sphericity, total sample size, degree of sample size imbalance and number of levels of the within-subjects factor. Consistent with previous research, the WJ and IGA procedures based on least squares estimates were not always robust to violations of the multisample sphericity assumption when the data were obtained from multivariate nonnormal distributions. Adoption of trimmed mean estimators resulted in dramatic improvements in the Type I error performance of the WJ procedure. These results suggest that it is possible to obtain tests of within-subjects effects in split-plot designs which are robust to the combined effects of assumption violations.

Trimmed Means in Split-Plot Repeated Measurement Designs

The repeated measurement (RM) design is one of the most common research paradigms adopted by educational and psychological researchers. Moreover, Lix, Kowalchuk, and Keselman (1996) found, in their methodological content analysis of education articles published in 1994; that split-plot designs, which contain both between-subjects (groups) and within-subjects (trials) factors, are overwhelming favoured by researchers. In fact, split-plot designs were represented in 84% of all articles in which RM designs were used. Furthermore, the authors determined that unbalanced designs, in which there are unequal numbers of observations in each group (or cell) of the design, were more popular than balanced designs.

It is strongly recommended that the traditional analysis of variance (ANOVA) F-test be avoided for testing within-subjects main and interaction effects in split-plot RM designs, particularly when the design is unbalanced, because of the extreme sensitivity of this approach to departures from the assumption of multisample sphericity (see e.g., Keselman, Lix, & Keselman, 1996). Moreover, although either a degrees of freedom (df) adjusted univariate approach or a conventional multivariate approach may sometimes offer robust tests of the within-subjects main effect in such designs, these tests are rarely appropriate for tests of the interaction effect due to their sensitivity to heterogeneity of the group (or cell) covariance matrices.

For these reasons, extensive research has been conducted on two alternative solutions which are not dependent on the assumption of multisample sphericity. These solutions are the approximate df multivariate Welch-James solution (WJ; see Johansen, 1980; Keselman, Carriere, & Lix, 1993) and the Improved General Approximation (IGA; see Algina, 1994; Huynh, 1978) procedure. Either approach can provide satisfactory results, assuming that the data are normally distributed and provided that minimum sample size requirements are adhered to (Algina & Oshima, 1994, 1995; Algina & Keselman, 1996b). While the minimum sample size requirements for the IGA procedure tend to be much smaller than for the WJ solution, the latter approach can, under certain conditions, afford researchers substantially greater power than the former. Thus research conducted to date indicates that one approach is not consistently superior to the other.

While the WJ and IGA procedures are robust to the effects of multisample sphericity when the data are normally distributed, neither approach can effectively control the Type I error rate when the data are obtained from nonnormal distributions. The underlying problem is that the usual (i.e., least squares) mean and variance (covariance), which are the basis for both of these procedures, are greatly influenced by the presence of extreme observations in a distribution of scores. In particular, the standard error of the usual mean can become seriously inflated when the underlying distribution has heavy tails. While a wide range of estimators have been proposed in the literature as replacements for the usual mean and variance (see Gross, 1976), the trimmed mean and Winsorized variance are the most intuitively appealing robust estimators because of their computational simplicity and good theoretical properties (Wilcox, 1995a). The standard error of the trimmed mean is less affected by departures from normality than the usual mean because extreme observations, that is, observations in the tails of a distribution, are censored or removed. Furthermore, as Gross (1976) notes, "the Winsorized variance is a consistent estimator of the variance of the corresponding trimmed mean" (p. 410). In computing the Winsorized variance, the most extreme observations are replaced with less extreme values in the distribution of scores.

Studies conducted within the context of other research paradigms, namely univariate one-way and factorial independent groups designs suggests that it is possible to obtain test procedures which are robust to the combined effects of variance heterogeneity and nonnormality by the adoption of trimmed means and Winsorized variances instead of the usual least squares estimates (e.g., Keselman, Lix, & Kowalchuk, 1996; Keselman, Kowalchuk, & Lix, 1996; Lix, Keselman, & Carriere 1997). In particular, the WJ solution has shown encouraging results in such situations. Moreover, Wilcox (1993) investigated the use of trimmed means and Winsorized variances (covariances) within the context of a single-group RM design when df-adjusted univariate procedures were used to test the within-subjects effect and also found substantial improvement in Type I error control in the presence of nonnormality.

In the present paper, we were primarily concerned with extending procedures for testing within-subjects main and interaction effects in split-plot RM designs to data-analytic conditions in which neither the assumptions of multivariate normality nor multisample sphericity hold. Therefore, the purpose of this investigation was to determine whether the use of trimmed means and Winsorized variances (covariances) with the WJ and IGA procedures can result in improved tests for mean equality under departures from multivariate normality.

Definition of Test Procedures

Consider the simplest split-plot RM design with a single between-subjects factor with $j = 1, \ldots, J$ levels and n_j observations at each j ($\Sigma_{j=1}^{J} n_j = N$) and a single within-subjects factor with $k = 1, \ldots, K$ levels. Thus $\mathbf{Y}_{ij} = [Y_{ij1}\ Y_{ij2} \ldots Y_{ijK}]$ for $i = 1, \ldots, n_j$, denotes the vector of scores associated with the *i*th observation in the *j*th group. It is assumed at the outset that the

Y_{ij}s are obtained from a multivariate normal distribution with mean vector μ_j and variance-covariance matrix Σ_j.

It is important to note, that when trimmed means are adopted instead of the usual least-squares means, the null hypotheses which are being tested relate to equality of the population trimmed means (i.e., μ_{tjk}s), not the usual means (i.e., μ_{jk}s). Therefore, under trimming and in the context of split-plot RM designs, the null hypothesis associated with the within-subjects main effect is

$$H_K: \mu_{t.1} = \mu_{t.2} = \ldots = \mu_{t.K},$$

while for the within-subjects interaction effect, the null hypothesis under investigation is

$$H_{JK}: \mu_{tjk} - \mu_{tj.} - \mu_{t.k} + \mu_{t..}, \text{ for all j, k.}$$

The computational formulas for the WJ and IGA procedures based on usual least squares estimates have been defined elsewhere, and thus will not be repeated here. In particular, the reader may consult Lix and Keselman (1995) and Algina and Oshima (1995) respectively for information concerning these procedures. As well, a copy of the SAS/IML program (SAS Institute, 1989) which may be used to compute the WJ solution for a split-plot design can be obtained from the paper by Lix and Keselman.

The trimmed means solution adopted in this paper is based on symmetric trimming. Hence, let $g_j = [\gamma_s n_j]$, where γ_s represents the proportion of observations that are to be trimmed in each tail of a distribution. The effective sample size for the *j*th group is $h_j = n_j - 2g_j$. The procedure used to compute the K-dimensional vector of trimmed means for the *j*th group was the same as that defined by Wilcox (1993) within the context of a single-group RM design (see pp. 67-68). Furthermore, Wilcox's computational formula for the

Winsorized variance-covariance matrix for the *j*th group was also adopted. Finally, it should be noted that the use of trimmed means and Winsorized variances (covariances) was based on the removal of 20 percent of the observations from each tail of the distribution of scores in each cell of the J x K data matrix for the split-plot RM design. This 20 percent rule is well-established (see Rosenberger & Gasko, 1983; Wilcox, 1994b) and is based in part on optimizing power for nonnormal as well as normal distributions (see Wilcox, 1994a).

Methodology

A Monte Carlo simulation study was undertaken to empirically evaluate the Type I error performance of the WJ solution to that of the IGA procedure for testing RM main and interaction effect hypotheses when either the usual mean and variance (covariance) or the trimmed mean and Winsorized variance (covariance) were employed as measures of central tendency and scale, respectively. These tests were investigated for a RM design containing a single between-subjects factor and a single within-subjects factor.

Seven variables were manipulated in the simulation study. These were: (a) number of levels of the within-subjects factor, (b) total sample size, (c) degree of group size equality/inequality, (d) degree of equality/inequality of the group variance-covariance matrices, (e) nature of the pairing of the group sizes and group covariance matrices, (f) degree of departure from the sphericity assumption, and (g) multivariate normality/nonnormality. All of these variables have been manipulated in previous investigations and were therefore selected based on the knowledge that they may influence the Type I error performance of both the WJ and IGA procedures. The one constant in the study was the number of levels of the between-subjects factor (J), which was set at three.

The number of levels of the within-subjects factor (K) was set at either four or eight. Total sample size (N) was set at either 84 or 105; these values were selected based on recommendations provided by Keselman et al. (1993) and Algina and Keselman (1996b) concerning minimum sample size requirements for the WJ procedure and bearing in mind that under the trimming rule adopted in this study, 20 percent of the observations will be removed from each tail of a distribution.

With respect to the third variable, two degrees of group size imbalance were selected such that a coefficient of variation of group size inequality (i.e., Δn_j) was approximately equal to .16 and .32 for each value of total sample size. Thus, for N = 84, the individual group sizes (i.e., n_js) were 22, 28, and 34 for $\Delta n_j \simeq .16$ and 17, 28, 39 for $\Delta n_j \simeq .32$. For N = 105, the n_js were 28, 35, 42 and 21, 35, 49, respectively for the two values of Δn_j. It is important to note that while it was of primary interest to examine Type I error performance in unbalanced designs, we also investigated conditions in which group sizes were equal (i.e., $\Delta n_j = 0$) for comparison purposes; thus for N = 84, each group contained 28 observations, while for N = 105, each group contained 35 observations.

The degree of heterogeneity of the group covariance matrices (i.e., Σ_js) was manipulated based on conditions investigated by Algina and Keselman (1996b), so that elements of the matrices were either in a 1:3:5 or 1:5:9 ratio. Thus, for each degree of heterogeneity of the group covariance matrices, the following conditions were evaluated: (a) equal group sizes and unequal covariance matrices, (b) unequal group sizes ($\Delta n_j \simeq .16$) and unequal covariance matrices, (c) unequal group sizes ($\Delta n_j \simeq .32$) and unequal covariance matrices. As well, both positive and negative pairings of group sizes and group covariance matrices were considered for the latter two conditions. A positive pairing refers to a condition in which the largest n_j is associated with the covariance matrix containing the largest element values, while a negative pairing refers to a condition in which the smallest n_j is associated with the covariance matrix containing the largest element values.

The degree of departure from the assumption of sphericity (ϵ) was also manipulated in this study, and ϵ assumed values of 1.0, .75, and .57. Algina and Keselman (1996a, 1996b), found that the performance of the WJ procedure varies slightly as a function of the value of ϵ. Readers will recall that when $\epsilon = 1.0$, sphericity is satisfied, and that for a split-plot design, this parameter can assume a lower bound of $\epsilon = 1/(K - 1)$.

Finally, the WJ and IGA procedures were compared when the simulated data were obtained from multivariate normal and multivariate nonnormal distributions. In particular, under nonnormality, two skewed distributions were considered since it has been established that both the IGA and WJ procedures are sensitive to the presence of skewed data. The first was the multivariate lognormal distribution with marginal distributions based on $Y_{ijk} = \exp(X_{ij})$ $(i = 1 ,\ldots, n_j; j = 1 ,\ldots, J; k = 1 ,\ldots, K)$, where $X_{ijk} \sim N(0, .25)$; this distribution has skewness and kurtosis values of 1.75 and 5.90, respectively. Algina and Oshima (1995) provide details of the method for simulating multivariate data having the properties of a lognormal distribution. Data for the second multivariate nonnormal distribution were generated using the algorithm provided by Vale and Maurelli (1983) which is based on the method of Fleishman (1978). The data were generated such that skewness and kurtosis values were 1.75 and 3.75, respectively. Each observation vector $\mathbf{Y}_{ij} = [Y_{ij1} \ldots Y_{ijK}]$ with mean vector $\mathbf{0}$ and covariance matrix $\boldsymbol{\Sigma}_j$ was obtained by first generating a vector of variates having a univariate standard normal distribution (i.e., $\mathbf{Z}_{ij}$) using the SAS (SAS Institute, 1989) generator RANNOR. A vector of constants, $\mathbf{w} = [a\ b\ c\ d]^T$ was obtained from Fleishman's Table 1 (pp. 524-525) to provide the desired degree of multivariate skewness and kurtosis. An intermediate covariance matrix (i.e., λ_j) was computed so that $\mathbf{Y}_{ij}$ would have the desired $\boldsymbol{\Sigma}_j$. Elements of this intermediate matrix were computed using Vale and Maurelli's Equation 11 (p. 467), which involves finding the roots of a third-degree polynomial; these roots were computed using the SAS/IML POLYROOT function (SAS Institute, 1989). Thus, the vector of univariate standard normal deviates was transformed to a vector of multivariate normal deviates via the Cholesky decomposition

$$\mathbf{Z}(\lambda)_{ij} = \mathbf{0} + \mathbf{L}_\lambda \mathbf{Z}^T_{ij},$$

where $\mathbf{Z}(\lambda)_{ij}$ is the vector of transformed variates, $\mathbf{0}$ is the zero vector, and $\mathbf{L}_\lambda$ is an upper triangular matrix of dimension K satisfying the equality $\mathbf{L}_\lambda^T\mathbf{L}_\lambda = \lambda_j$. Next, each element of $\mathbf{Y}_{ij}$ was obtained by computing the zero through third powers of the corresponding element of $\mathbf{Z}(\lambda)_{ij}$, so that $\mathbf{Z}(\lambda)_{ijk} = [1\ Z(\lambda)_{ijk}\ Z(\lambda)^2_{ijk}\ Z(\lambda)^3_{ijk}]$ represents the vector of powers for the kth component of $\mathbf{Z}(\lambda)_{ij}$ (k = 1 ,..., K). From this, $Y_{ijk} = \mathbf{Z}(\lambda)_{ijk}\mathbf{w}$.

The simulation program was written in the SAS/IMl (SAS Institute, 1989) programming language. Five thousand replications were generated for each condition and Type I error rates were collected using a .05 significance level.

Results

A quantitative measure of robustness suggested by Bradley (1978) was used to evaluate the Type I error performance of the WJ and IGA procedures for least squares and trimmed means estimators. According to this criterion, in order for a test to be considered robust, its empirical Type I error rate ($\hat{\alpha}$) must be contained in the interval $.5\alpha \leq \hat{\alpha} \leq 1.5\alpha$. Therefore, for the five percent level of significance adopted in this study, a test was considered to be robust in a particular condition if its empirical Type I error rate was contained in the interval $.025 \leq \hat{\alpha} \leq .075$. Correspondingly, a test was considered to be nonrobust if, for a particular condition, its Type I error rate was not contained in this interval. In all subsequent tables, values not falling within the bounds of Bradley's criterion are given in boldface.

Multivariate Normal Data

K = 4

The results associated with the least squares and trimmed methods of estimation for the multivariate normal distribution when K = 4 are contained in Table 1. Only the results associated with the minimum value of N (i.e., N = 84) are provided, as very similar results were obtained for N = 105. As is apparent from this table, when the usual least squares method of estimation was used in conjunction with the WJ and IGA procedures, Type I error rates were maintained within the bounds of Bradley's (1978) liberal criterion under both degrees of covariance heterogeneity (i.e., 1:3:5 and 1:5:9). Furthermore, ϵ had little effect on the magnitude of error rates, and thus error rates were well-controlled even under departures from sphericity. Thus, as expected, the WJ and IGA procedures were robust to departures from the assumption of multisample sphericity when the data were normally distributed and least squares estimates were used.

Insert Table 1 about here

When trimmed means and Winsorized variances (covariances) were used in conjunction with the WJ and IGA procedures and the data were normally distributed, the rates of Type I error were also well-controlled and were very similar, although in some cases slightly lower, than those obtained when least squares estimates were used. Across all of the conditions considered in Table 1, the average Type I error rates for the WJ main and interaction effect tests were 4.99% and 5.21%, respectively for least squares estimates, and for trimmed means estimates they were 4.33% and 4.29%, respectively. For the IGA solution, the corresponding results for least squares estimates were 4.93% and 4.86%, respectively, and for trimmed estimates were 4.84% and 4.79%, respectively. These results also indicate that there were very minor differences in error rates between the WJ and IGA procedures for tests of within-subjects main and interaction effects under a multivariate normal distribution

K = 8

Table 2 contains the results associated with the least squares and trimmed mean estimators for the multivariate normal distribution when the number of levels of the within-subjects factor was increased to eight and N = 84. When least squares estimates were employed, and covariance matrices were in a ratio of 1:3:5, only the WJ test of the interaction effect produced Type I error rates which were not contained within the bounds of Bradley's (1978) liberal criterion. In particular, liberal rates occurred for negative pairings of group sizes and covariance matrices (i.e., conditions B^- and C^-). This same trend existed under the more extreme pattern of covariance heterogeneity (i.e., 1:5:9), although condition A also resulted in a slightly liberal value (i.e., 7.60%) when $\epsilon = 1.0$. As well, when $\epsilon = .75$, the WJ test of the within-subjects main effect produced a single value which only slightly exceeded the upper bound of Bradley's (1978) liberal criterion under condition C^- (7.56%).

Insert Table 2 about here

When trimmed means and Winsorized variances (covariances) were used with the WJ procedure, error rates were generally lower than when least squares estimates were employed. However, four liberal values were obtained for the WJ trimmed mean approach represented in Table 2. These were obtained for the test of the interaction effect when $\epsilon < 1.0$ and negative pairings were investigated under the more extreme degree of group size imbalance (i.e., Condition C^-).

The Type I error rates associated with the IGA procedure were well-controlled across all of the conditions reported in Table 2. Furthermore, there was very little difference between those values obtained for least squares and trimmed means estimates for either the main or interaction effects.

The rates of Type I error associated with the normal distribution for N = 105 and K = 8 have not been tabled, as they were largely consistent with those reported in Table 2. Specifically, while the WJ test of the interaction effect tended to result in liberal values for negative pairings of group sizes and covariance matrices when least squares estimates were adopted. These values were, however, lower than those reported in Table 2. Hence, the maximum value obtained for the WJ test of the interaction was 8.56%. All of the Type I error rates associated with the IGA procedure under least squares estimation were contained within the bounds of Bradley's (1978) liberal criterion Moreover, the results obtained for trimmed means when N = 105 revealed that all of the rates associated with the WJ and IGA procedures were within the bounds of Bradley's criterion.

Multivariate Nonnormal Data

K = 4

Table 3 contains the results associated with the lognormal distribution for K = 4 and N = 84. When the least squares method of estimation was adopted, liberal rates occurred under only the most extreme degree of departure from the sphericity assumption (i.e., ϵ = .57). When covariance matrices were in a ratio of 1:3:5, the WJ test of the within-subjects main effect was liberal under condition C^-, while the WJ test of the interaction effect was liberal under conditions A (7.84%), B^- (8.28%), and C^- (9.34%). When the degree of covariance heterogeneity increased, the WJ test of the interaction produced liberal results across all five of the investigated conditions when ϵ = .57.

Insert Table 3 about here

Error rates for the IGA procedure were always contained within the bounds of Bradley's (1978) criterion when the data followed a lognormal distribution and least squares estimates were employed. However, values tended to be slightly more extreme than those obtained for the multivariate normal distribution; the minimum and maximum Type I error rates reported in Table 3 for the IGA procedure are 3.26% and 6.66%, respectively while in comparison, the respective values obtained for normally distributed data are 4.32% and 5.74% (see Table 1).

Finally, as Table 3 reveals, when trimmed mean estimates were employed for lognormal data, Type I error rates were well controlled for both the WJ and IGA procedures.

The results obtained for the multivariate nonnormal distribution generated via the Vale and Maurelli (1983) approach, with skewness of 1.75 and kurtosis of 3.75, have not been tabled due to the similarities in results for N = 84. Specifically, no liberal values were obtained for values of $\epsilon > .57$ for the WJ or IGA solutions when either least squares or trimmed means estimators were adopted. Under the most extreme departure from sphericity, however, the WJ test of the interaction effect did produce a number of liberal values when least squares estimates were employed; the maximum value was 11.38%. The WJ test of the main effect was liberal for both degrees of covariance heterogeneity under condition C^- for $\epsilon = .57$, while the IGA test of the within-subjects main effect was also slightly liberal (7.52%) for this condition when covariance matrices were in a 1:5:9 ratio. The results obtained for trimmed means revealed that error rates were always contained within the bounds of Bradley's criterion. This same finding holds for N = 105 when the data were generated from either of the investigated nonnormal distributions.

K = 8

For K = 8, the empirical percentages of Type I error for the multivariate lognormal data which were obtained when N = 84 can be found in Table 4. While the WJ procedure produced numerous liberal values when least squares estimates were employed, the majority of these were obtained for values of $\epsilon < 1.0$. In fact, for $\epsilon = .57$, the WJ tests of the within-subjects main and interaction effect were always liberal, with error rates as high as 21.68% for the test of the interaction effect. While the IGA test provided generally good Type I error control, a single conservative value of 2.38% was obtained for least squares estimates (ϵ = 1.0; condition C^-; 1:5:9).

Insert Table 4 about here

The use of trimmed means resulted in a dramatic improvement in the Type I error performance of the WJ procedure. As Table 4 reveals, only three of the 60 error rates associated with this approach were not contained within the bounds of Bradley's (1978) liberal criterion. All three of these values were obtained for the test of the interaction effect under the C^- condition when $\epsilon < 1.0$. Consistent with previously tabled findings for the IGA procedure, none of the rates associated with trimmed mean estimators fell outside of Bradley's bounds.

Finally, for comparison purposes, the results associated with the lognormal distribution for K = 8 and N = 105 are reported in Table 5. As anticipated and consistent with previous research (e.g., Keselman et al. 1993), increasing total sample size does result in somewhat lower error rates for the WJ procedure, particularly for tests of the interaction effect. However, many liberal values still remain for $\epsilon < 1.0$ for both the within-subjects main and interaction effects. At the same time, none of the WJ values associated with trimmed mean estimators were liberal.

Insert Table 5 about here

The findings reported in Tables 4 and 5 are consistent with the results obtained for the Vale and Maurelli (1983) distribution with skewness of 1.75 and kurtosis of 3.75 (untabled). That is, while the WJ solution based on least squares estimators resulted in a number of liberal values, particularly when $\epsilon < 1.0$, the IGA procedure did not. The IGA solution with least squares means did, however, become conservative for the test of the within-subjects interaction effect under the C^- condition when $\epsilon = 1.0$ and covariance matrices were in a 1:5:9 ratio for N = 84. The WJ and IGA solutions based on trimmed means always produced error rates which were contained within the bounds of Bradley's (1978) criterion.

Discussion and Conclusions

This investigation compared four procedures which can be used to test within-subjects effects in split-plot repeated measurement designs when the assumption of multisample sphericity is not satisfied and data are obtained from multivariate nonnormal distributions. Specifically, we compared the WJ approximate df multivariate solution as given by Johansen (1980) to the IGA method based on the work of Huynh (1978) and Algina (1994). When utilizing the usual mean and variance (covariance), these two procedures test equality of the population means, while the use of trimmed means and Winsorized variances (covariances) results in tests of equality of population trimmed means.

Based on the findings of this simulation study, it is apparent that it is possible to obtain solutions which are robust to the combined violations of the multisample sphericity and multivariate normality assumptions for testing within-subjects main and interaction effects in split-plot repeated measurement designs. Specifically, the adoption of trimmed means resulted in substantial improvements in the operating characteristics of the WJ multivariate solution for the investigated conditions. While the IGA procedure was largely robust to the assumption violations considered in this study, in those few instances where either liberal or conservative Type I error rates were obtained under least squares estimates, the performance of this procedure could be improved through the use of robust estimates. The reader should also bear in mind that the total sample size conditions investigated in this study were based on recommendations provided in previous studies concerning the minimum sample size requirements for the WJ solution. Hence, with smaller sample sizes, the performance of the IGA procedure might be less than optimal, and thus the adoption of trimmed means might prove to be significantly more advantageous. Moreover, the results indicate that robust test solutions may be obtained in both balanced designs and unbalanced designs.

While the results of this study are highly encouraging, they also lead to questions regarding performance with respect to statistical power. In particular, neither the WJ nor IGA tests based on trimmed means can be recommended in practice unless their performance, as compared with procedures based on least squares means, is similar when the normality assumption is satisfied. Furthermore, comparisons of the two procedures under trimmed means estimation is necessary to make recommendations concerning the adoption of one approach over the other under conditions of nonnormality. While Algina and Keselman (1996a) have suggested that in comparisons of the power of the WJ and IGA procedures, the WJ procedure almost always offers greater power advantages when least squares estimates are adopted. However, it is not known whether this findings can be generalized to the use of trimmed mean estimators.

Table 6 provides preliminary empirical power results associated with tests of the within-subjects main effect for the WJ and IGA procedures when K = 4. These results were obtained for a single mean configuration (i.e., $0\ -\mu\ \mu\ 0$) identified by Algina and Keselman (1996a) as highlighting the differences in power performance between the WJ and IGA solutions. The results reported in Table 6 have been averaged across the two sample size conditions (i.e., N = 84 and N = 105) and two effect sizes of a moderate magnitude.

Insert Table 6 about here

The data reported in Table 6 pertaining to the normal distribution allows for a comparison of the WJ and IGA procedures when using least squares and trimmed mean estimation procedures. Only the results associated with the more extreme degree of variance heterogeneity are provided, however both solutions resulted in robust Type I error rates under this degree of heterogeneity when the data were normally distributed. While comparisons are difficult to make for $\epsilon = .57$ for the WJ test due to the fact that power tends to reach an upper bound, the remaining results indicate that while power associated with least squares estimates is always larger, the differences in power between least squares and trimmed means estimators are, in general, not substantial. For example, the average value obtained for the WJ test of the within-subjects main effect when $\epsilon = 1.0$ and the data are normally distributed is 36.74% for least squares estimation and 29.41% for trimmed means estimation. Corresponding values for the IGA test when $\epsilon = 1.0$ are 38.33% and 33.14%, respectively.

The data reported in Table 6 for the lognormal distribution allow for a comparison of the WJ and IGA procedures when trimmed mean estimators are adopted. The results reported here mirror those reported by Algina and Keselman (1996a) for least squares estimates. That, is, while the IGA procedure is sometimes slightly more powerful than the WJ solution (i.e., $\epsilon = 1.0$), under departures from the assumption of sphericity the WJ solution can be substantially more powerful than the IGA procedure. To illustrate, for condition A when $\epsilon = .75$, the power for the WJ main effect test is 94.52%, while the IGA rate is approximately 20% lower, at 74.68%.

These preliminary power results and the previous Type I error results suggest that the Welch-James procedure can provide a robust and powerful test of the within-subjects main effect in split-plot designs when trimmed mean estimators are adopted. Further power results will help to clarify any differences that might exist between the Welch-James and Improved General Approximation procedures for tests of the within-subjects interaction effect.

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Table 1. Empirical Percentages of Type I Error for Least Squares and Trimmed Means Estimation (Normal Data; K = 4; N = 84)

	$\epsilon = 1.0$				$\epsilon = .75$				$\epsilon = .57$			
Condition	WJ_M	IGA_M	WJ_I	IGA_I	WJ_M	IGA_M	WJ_I	IGA_I	WJ_M	IGA_M	WJ_I	IGA_I
Least Squares Estimation; 1:3:5												
A	4.74	4.50	5.06	4.46	5.20	4.50	5.62	5.26	5.06	4.90	4.88	4.32
B^+	4.46	4.50	5.02	4.56	4.74	4.72	4.88	4.86	5.20	4.76	5.04	5.38
B^-	5.38	4.94	5.10	4.88	4.66	4.54	5.20	5.18	5.08	4.88	5.08	5.16
C^+	5.12	4.86	5.00	4.84	4.60	4.44	4.90	4.78	4.84	5.32	5.82	5.08
C^-	5.04	4.82	5.40	4.36	5.66	5.24	5.90	5.12	5.26	5.44	5.80	5.24
Trimmed Means Estimation; 1:3:5												
A	4.08	4.32	3.90	4.80	4.16	4.74	4.08	4.98	4.64	4.86	4.42	4.32
B^+	4.54	4.92	3.98	4.82	4.24	4.38	3.70	4.58	4.60	5.08	4.52	5.04
B^-	4.52	4.76	4.22	4.58	3.86	4.64	3.92	4.98	4.40	5.14	4.82	5.50
C^+	4.70	4.78	4.70	5.26	4.38	4.30	3.72	4.38	4.58	5.02	4.42	5.00
C^-	3.82	4.74	3.92	4.24	4.40	5.26	4.26	4.80	4.64	5.46	5.34	5.34
Least Squares Estimation; 1:5:9												
A	4.88	5.04	4.74	4.46	5.10	5.74	6.00	5.00	5.14	5.30	5.20	4.76
B^+	5.20	5.08	5.58	4.42	4.90	5.38	5.20	4.50	5.08	4.48	4.84	4.96
B^-	5.22	4.76	5.28	4.96	5.38	5.06	5.86	5.08	4.70	4.78	5.50	4.84
C^+	4.76	4.62	4.74	4.56	4.96	4.74	4.98	4.76	4.36	4.82	5.44	5.14
C^-	4.78	4.46	5.62	4.36	4.50	5.34	5.42	5.04	5.16	5.64	4.80	5.30
Trimmed Means Estimation; 1:5:9												
A	4.34	4.76	3.94	4.40	4.28	5.06	4.22	4.64	4.28	4.84	4.36	4.38
B^+	4.56	4.96	4.38	4.72	3.98	4.90	3.94	4.50	4.80	4.84	4.94	5.24
B^-	4.30	4.82	4.18	4.84	4.28	4.70	4.58	4.92	4.52	5.10	4.64	5.08
C^+	4.58	4.58	4.14	4.48	3.98	4.70	3.98	4.40	4.22	4.26	4.88	5.26
C^-	3.80	4.44	3.98	4.54	3.92	4.82	4.46	5.26	4.66	5.74	5.64	5.60

Note: WJ_M = WJ main effect; IGA_M = IGA main effect; WJ_I = WJ interaction effect; IGA_I = IGA interaction effect; values in boldface fall outside the range of 2.50 - 7.50; Conditions A = equal group sizes/unequal covariance matrices; B = moderately unequal group sizes and unequal covariance matrices, either positively (+) or negatively (-) paired; C = extremely unequal group sizes and unequal covariance matrices, either positively (+) or negatively (-) paired.

Table 2. Empirical Percentages of Type I Error for Least Squares and Trimmed Means Estimation (Normal Data; K = 8; N = 84)

	ϵ = 1.0				ϵ = .75				ϵ = .57			
Condition	WJ_M	IGA_M	WJ_I	IGA_I	WJ_M	IGA_M	WJ_I	IGA_I	WJ_M	IGA_M	WJ_I	IGA_I
Least Squares Estimation; 1:3:5												
A	5.12	5.04	6.24	3.86	5.04	4.52	6.44	4.92	5.38	4.66	7.24	4.96
B^+	5.56	4.86	6.44	4.36	5.38	5.00	6.42	5.24	4.68	4.28	6.04	4.80
B^-	5.54	4.16	**7.84**	3.72	5.86	4.68	**7.94**	4.36	5.28	4.86	7.18	4.34
C^+	4.96	4.66	5.84	4.68	4.58	5.28	5.94	5.08	4.86	4.90	6.24	4.80
C^-	6.64	3.78	**10.34**	3.54	6.58	4.34	**10.60**	4.06	6.40	4.50	**10.88**	4.74
Trimmed Means Estimation; 1:3:5												
A	3.80	4.66	4.00	4.06	3.24	4.74	4.40	4.58	3.98	4.58	4.94	4.48
B^+	3.90	4.82	4.06	4.50	3.30	4.44	4.10	4.24	3.26	4.80	3.96	4.38
B^-	3.80	5.14	4.88	4.10	3.20	4.26	5.42	4.16	3.60	4.54	5.34	4.12
C^+	3.56	4.40	3.92	4.64	3.66	4.68	4.26	4.84	3.82	5.02	3.86	4.38
C^-	4.14	3.92	6.86	3.62	4.32	4.44	**7.52**	3.86	4.46	4.34	**7.88**	3.96
Least Squares Estimation; 1:5:9												
A	5.42	4.56	**7.60**	4.80	5.70	5.14	6.64	4.60	5.22	5.00	6.72	4.84
B^+	4.90	4.46	6.70	4.16	4.92	5.16	6.90	5.20	4.84	4.48	6.30	5.38
B^-	6.58	4.78	**8.26**	4.14	5.36	4.54	**7.88**	4.42	6.10	5.30	**8.08**	5.40
C^+	4.90	4.62	6.40	4.26	4.78	5.50	6.64	4.96	5.24	5.56	6.06	4.60
C^-	7.06	4.28	**10.92**	3.52	**7.56**	4.72	**11.50**	4.52	6.46	4.62	**9.96**	4.50
Trimmed Means Estimation; 1:5:9												
A	3.26	4.48	4.14	3.90	3.76	4.78	4.46	4.20	3.32	4.66	4.52	4.28
B^+	3.44	4.96	3.60	4.08	3.38	4.62	3.94	4.34	3.48	4.46	4.16	4.80
B^-	3.82	5.00	4.92	4.10	3.16	4.04	5.26	3.94	3.80	4.98	6.02	4.94
C^+	3.64	5.04	3.78	4.34	3.66	5.32	4.02	4.10	3.36	5.32	4.52	4.70
C^-	3.98	4.46	6.48	3.66	4.38	3.92	**8.28**	3.70	4.56	4.40	**8.06**	4.18

Note: See the note for Table 1.

Table 3. Empirical Percentages of Type I Error for Least Squares and Trimmed Means Estimation (Lognormal Data; K = 4; N = 84)

	$\epsilon = 1.0$				$\epsilon = .75$				$\epsilon = .57$			
Condition	WJ_M	IGA_M	WJ_I	IGA_I	WJ_M	IGA_M	WJ_I	IGA_I	WJ_M	IGA_M	WJ_I	IGA_I
Least Squares Estimation; 1:3:5												
A	4.98	4.00	5.32	3.76	5.38	4.86	6.02	5.00	6.70	5.90	**7.84**	5.02
B^+	4.58	4.02	4.88	3.82	5.24	5.10	5.48	4.64	6.72	5.60	7.02	5.16
B^-	5.62	4.36	5.12	3.96	5.54	4.98	5.96	4.72	7.34	5.78	**8.28**	5.70
C^+	5.16	4.24	5.20	4.34	5.48	4.90	5.30	5.28	6.04	5.94	6.84	5.12
C^-	5.50	3.82	5.54	3.26	5.74	5.42	6.28	4.94	**7.76**	6.66	**9.34**	6.06
Trimmed Means Estimation; 1:3:5												
A	3.92	3.96	3.58	4.02	3.86	4.46	3.88	4.78	4.78	4.86	4.58	4.42
B^+	4.38	4.58	3.72	3.18	3.52	4.16	3.54	4.20	4.54	4.92	3.94	5.08
B^-	4.18	4.44	4.06	4.14	3.68	4.44	3.66	4.68	4.64	5.08	5.18	5.48
C^+	4.54	4.56	4.28	4.78	4.02	4.28	3.16	4.20	4.46	4.76	4.30	4.72
C^-	3.80	4.16	3.94	3.46	3.62	4.50	3.74	4.56	4.52	5.48	5.88	5.46
Least Squares Estimation; 1:5:9												
A	5.40	4.22	5.44	3.88	5.36	5.24	6.46	4.72	7.14	5.84	**9.18**	5.22
B^+	5.52	4.70	5.76	3.98	5.48	5.30	5.80	4.54	6.68	5.74	**8.34**	4.92
B^-	5.16	3.64	5.94	3.68	6.04	5.14	6.78	4.74	6.54	5.92	**9.58**	5.60
C^+	5.00	4.48	5.34	4.16	5.74	5.12	5.56	4.64	6.36	5.36	**7.86**	5.06
C^-	4.80	4.12	6.10	3.56	5.48	4.80	6.86	4.38	7.26	6.84	**9.44**	6.42
Trimmed Means Estimation; 1:5:9												
A	4.04	4.14	3.72	3.90	3.78	4.82	3.82	4.46	4.20	4.82	4.92	4.90
B^+	4.50	4.94	4.02	4.50	3.92	4.64	3.66	4.38	4.82	5.00	5.38	4.86
B^-	4.40	4.36	4.14	4.18	3.98	4.78	4.14	4.66	4.28	5.00	5.66	4.88
C^+	4.10	4.14	3.80	4.14	3.80	4.40	3.56	4.28	4.10	4.58	5.14	5.14
C^-	3.54	3.84	3.74	3.80	3.50	4.38	4.22	4.64	4.40	5.44	5.26	5.48

Note: See the note for Table 1.

Table 4. Empirical Percentages of Type I Error for Least Squares and Trimmed Means Estimation (Lognormal Data; K = 8; N = 84)

Condition	ϵ = 1.0				ϵ = .75				ϵ = .57			
	WJ_M	IGA_M	WJ_I	IGA_I	WJ_M	IGA_M	WJ_I	IGA_I	WJ_M	IGA_M	WJ_I	IGA_I
Least Squares Estimation; 1:3:5												
A	5.62	4.16	6.90	2.96	6.38	4.22	**9.54**	4.04	**8.94**	4.88	**12.70**	4.16
B^+	5.56	4.14	7.32	3.54	6.14	4.36	**8.08**	4.42	**9.12**	4.52	**10.76**	4.34
B^-	6.42	3.54	**9.22**	2.88	**7.58**	3.70	**11.70**	3.18	**10.28**	4.84	**14.66**	3.74
C^+	5.64	4.46	6.80	3.74	6.00	4.56	7.32	4.36	**8.08**	4.72	**9.08**	4.12
C^-	7.28	3.10	**11.46**	2.70	**8.56**	3.76	**14.80**	3.44	**11.58**	4.02	**19.66**	4.26
Trimmed Means Estimation; 1:3:5												
A	3.48	4.10	3.62	3.86	2.92	4.48	3.68	4.32	4.20	4.52	5.04	4.10
B^+	4.06	4.44	3.88	3.82	3.18	4.20	3.34	3.88	3.20	4.08	3.96	3.92
B^-	3.72	4.54	4.56	3.80	3.12	3.72	4.74	3.48	3.52	4.30	5.72	3.98
C^+	3.50	4.06	3.24	4.24	3.52	4.36	3.50	4.60	3.82	4.82	3.60	4.10
C^-	3.98	3.56	6.00	3.14	3.82	4.12	6.60	3.40	4.20	3.74	**8.40**	3.40
Least Squares Estimation; 1:5:9												
A	5.50	4.00	**8.34**	3.76	**7.96**	4.32	**10.60**	3.74	**10.02**	4.78	**15.72**	4.40
B^+	5.50	3.78	7.08	3.52	6.80	4.66	**10.38**	4.16	**8.72**	4.60	**15.04**	4.82
B^-	6.68	3.90	**9.56**	3.26	**7.88**	3.84	**11.96**	3.38	**10.68**	4.54	**17.66**	4.52
C^+	5.88	4.12	7.02	3.72	7.16	5.14	**8.48**	3.98	**8.60**	5.68	**12.06**	4.84
C^-	**7.86**	3.06	**12.02**	**2.38**	**9.54**	3.90	**16.46**	3.62	**12.54**	4.26	**21.68**	4.26
Trimmed Means Estimation; 1:5:9												
A	3.08	4.02	4.04	3.44	3.56	4.60	4.10	3.96	3.50	4.46	5.34	3.62
B^+	3.30	4.26	3.56	3.82	3.18	4.30	3.54	3.94	3.40	4.00	4.84	4.52
B^-	3.56	4.40	4.54	3.68	2.92	3.50	5.06	3.22	3.58	4.26	6.22	4.36
C^+	3.70	4.52	3.46	4.08	3.40	4.90	3.66	3.86	3.56	4.94	4.36	4.60
C^-	3.66	3.84	6.30	3.30	3.86	3.60	**7.74**	3.18	4.38	4.14	**8.80**	3.78

Note: See the note for Table 1.

Table 5. Empirical Percentages of Type I Error for Least Squares and Trimmed Means Estimation (Lognormal Data; K = 8; N = 105)

Condition	$\epsilon = 1.0$				$\epsilon = .75$				$\epsilon = .57$			
	WJ_M	IGA_M	WJ_I	IGA_I	WJ_M	IGA_M	WJ_I	IGA_I	WJ_M	IGA_M	WJ_I	IGA_I
Least Squares Estimation; 1:3:5												
A	5.70	4.30	6.80	3.32	6.06	4.72	**8.26**	4.02	**8.54**	4.76	**11.10**	3.96
B^+	5.40	4.30	6.28	4.04	6.68	4.60	6.86	4.36	**7.72**	5.18	**9.64**	4.50
B^-	5.60	3.34	7.18	2.96	7.44	4.24	**9.88**	3.72	**9.62**	4.72	**12.58**	4.70
C^+	5.18	4.68	3.34	4.00	6.32	4.48	7.24	4.00	**8.14**	5.50	**9.02**	5.08
C^-	6.68	3.86	**9.50**	2.64	**7.78**	3.78	**11.04**	3.66	**11.74**	4.38	**16.56**	3.76
Trimmed Means Estimation; 1:3:5												
A	3.60	4.26	3.36	3.78	3.12	3.96	3.30	3.98	3.84	4.30	3.98	4.02
B^+	3.44	4.30	2.86	4.32	3.30	4.06	3.18	4.20	3.62	4.70	3.62	4.58
B^-	3.34	4.16	3.46	3.40	3.44	4.32	3.86	3.66	4.08	4.10	4.70	4.26
C^+	3.82	5.00	2.90	4.30	3.92	4.64	3.06	4.22	3.76	5.04	3.88	4.58
C^-	3.64	4.54	4.64	3.44	2.86	3.98	4.42	3.68	3.74	4.10	5.94	3.90
Least Squares Estimation; 1:5:9												
A	5.64	3.48	7.40	3.30	6.24	3.84	**8.88**	4.18	**9.12**	4.36	**14.18**	4.28
B^+	5.32	4.26	7.38	4.00	7.04	4.54	**8.62**	3.74	**7.74**	4.28	**12.08**	4.62
B^-	6.72	3.66	**8.74**	3.00	**7.56**	4.66	**10.90**	3.74	**9.34**	4.20	**14.66**	4.30
C^+	5.14	3.74	6.14	4.28	6.34	4.62	**8.26**	4.34	**7.64**	4.62	**10.82**	4.80
C^-	6.72	3.40	**10.10**	2.58	**8.08**	3.96	**12.52**	4.06	**11.04**	4.28	**17.32**	4.44
Trimmed Means Estimation; 1:5:9												
A	3.24	4.24	3.34	4.10	3.56	4.34	3.80	3.56	3.82	4.90	4.30	3.64
B^+	3.28	4.64	3.40	4.36	3.94	4.46	3.66	3.94	4.02	4.58	3.96	4.14
B^-	3.92	4.56	3.76	3.62	3.84	4.34	4.32	4.16	3.54	4.34	5.08	4.00
C^+	3.00	3.88	3.32	4.26	3.44	4.06	3.44	4.34	3.64	4.34	4.14	4.28
C^-	3.52	3.78	4.64	3.22	2.86	3.66	4.46	3.38	4.18	3.70	6.42	3.80

Note: See the note for Table 1.

Table 6. Empirical Percentages of Power for Least Squares and Trimmed Means Estimation (K = 4; Averaged Across N)

	ϵ = 1.0		ϵ = .75		ϵ = .57	
Condition	WJ_M	IGA_M	WJ_M	IGA_M	WJ_M	IGA_M
Normal Data; Least Squares Estimation; 1:5:9						
A	37.23	38.67	92.96	64.47	99.72	56.54
B^+	42.58	44.24	96.14	72.79	99.89	67.09
B^-	30.78	32.52	86.89	53.01	98.51	45.03
C^+	48.04	49.48	97.59	79.03	99.89	74.02
C^-	25.05	26.75	75.20	40.52	95.33	34.29
Normal Data; Trimmed Means Estimation; 1:5:9						
A	29.37	32.85	80.56	50.85	96.62	44.40
B^+	34.78	38.27	87.06	59.69	98.59	52.88
B^-	24.37	28.28	69.99	40.81	91.72	35.66
C^+	40.06	42.82	91.24	65.03	99.32	58.74
C^-	18.46	23.45	54.08	30.36	80.66	27.45
Lognormal Data; Trimmed Means Estimation; 1:3:5						
A	43.91	47.40	94.52	74.68	99.63	67.47
B^+	51.45	54.53	96.90	82.04	99.91	75.93
B^-	37.92	41.90	88.54	63.98	98.44	56.61
C^+	54.94	58.12	97.97	85.80	99.98	81.00
C^-	28.14	33.42	77.23	50.81	94.19	44.54
Lognormal Data; Trimmed Means Estimation; 1:5:9						
A	40.21	43.70	91.63	68.86	99.26	61.50
B^+	47.21	50.48	95.52	77.97	99.75	70.94
B^-	33.25	36.86	83.79	57.96	97.25	50.66
C^+	53.43	56.11	97.05	82.94	99.92	76.79
C^-	27.66	30.05	70.37	44.22	90.95	39.47

Note: See the note for Table 1.

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